LAWS GOVERNING THE DISTRIBUTION OF HEAT IN BODIES OF FINITE DIMENSIONS AND THE APPLICATION OF THESE LAWS TO RADIATION HEATING

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Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 2, pp. 268–271, 1968

UDC 536.2:536.3

We consider the possibilities of applying the laws governing the distribution of heat in bodies of finite dimensions to the calculation of the heating of bodies by radiative heat.

The temperature field developed in bodies of finite dimensions assumes the particularly interesting property of being made to conform by the temperature distribution to the length of the coordinate axes or to the body surface. The form of this coordination relationship is governed by the conditions under which the process takes place. If the mathematical description of the phenomenon permits us to seek the solution of the problem in the form of the product of the functions

$$[1-\Theta(X;\ Y;\ \mathrm{Fo})] = U(X;\ \mathrm{Fo})\,V(Y;\ \mathrm{Fo}),$$

the coordination relationship is given strictly by

$$[1-\Theta(X;\ Y;\ \mathrm{Fo})] = \frac{[1-\Theta(0;\ Y;\ \mathrm{Fo})][1-\Theta(X;\ 0;\ \mathrm{Fo})]}{[1-\Theta(0;\ 0;\ \mathrm{Fo})]}, \qquad (1)$$

$$[1-\Theta(X;\ Y;\ \mathrm{Fo})] = \frac{[1-\Theta(1;\ Y;\ \mathrm{Fo})][1-\Theta(X;\ 1;\ \mathrm{Fo})]}{[1-\Theta(1;\ 1;\ \mathrm{Fo})]} \qquad (2)$$

and it can therefore be treated as the law governing the relationship between the temperatures, or what is the same, as the law governing the distribution of heat in bodies of finite dimensions. If the mathematical description of the phenomenon permits us to seek the solution of the problem in the form of the sum of the functions

$$\vartheta(X;\ Y;\ \mathrm{Fo}) = P(X;\ \mathrm{Fo}) + D(Y;\ \mathrm{Fo}),$$

the coordination relationship is also found strictly as

$$\vartheta(X;\ Y;\ \mathrm{Fo}) = \vartheta(0;\ Y;\ \mathrm{Fo}) + \vartheta(X;\ 0;\ \mathrm{Fo}) - \vartheta(0;\ 0;\ \mathrm{Fo}), \qquad (3)$$

$$\vartheta(X;\ Y;\ \mathrm{Fo}) = \vartheta(1;\ Y;\ \mathrm{Fo}) + \vartheta(X;\ 1;\ \mathrm{Fo}) - \vartheta(1;\ 1;\ \mathrm{Fo}) \qquad (4)$$

and can be treated as the law governing the relationship between temperatures or governing the distribution of heat in bodies of finite dimensions.

The first form for the heat-distribution law in bodies of finite dimensions was derived in the form of (1) and (2) in reference [1]. The second form of the law was established in the form of (3) and (4) in [2]. Each of these laws is remarkable in that neither incorporates the values for the thermophysical characteristics of the materials. Direct measurement of the temperature distributions over the coordinate axes or the surface of the body therefore makes it possible to calculate indirectly the temperature field within the body without first knowing the values of the thermal conductivity, heat capacity, or density of the material. As an example of a case corresponding to the first form of the heat-distribution law we can cite the regular period for the convective heating of a beam of rectangular or square cross section. As an example of the conditions subject to the second form of the distribution law we can offer the process of heating a beam of rectangular or square cross section by a continuous flow of heat [3 and 4]:

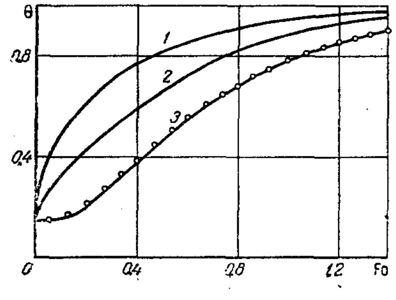
$$\frac{\partial\vartheta(X;\ Y;\ \mathrm{Fo})}{\partial\mathrm{Fo}} = \frac{\partial^2\vartheta(X;\ Y;\ \mathrm{Fo})}{\partial X^2} + \frac{R_1^2}{R_2^2}\frac{\partial^2\vartheta(X;\ Y;\ \mathrm{Fo})}{\partial Y^2}, \qquad (5)$$

$$\frac{\partial\vartheta(0;\ Y;\ \mathrm{Fo})}{\partial X} = \frac{\partial\vartheta(X;\ 0;\ \mathrm{Fo})}{\partial Y} = 0, \qquad (6)$$

$$\frac{\partial\vartheta(1;\ Y;\ \mathrm{Fo})}{\partial X} = \mathrm{Ki}_1;\quad \frac{\partial\vartheta(X;\ 1;\ \mathrm{Fo})}{\partial Y} = \mathrm{Ki}_2, \qquad (7)$$

$$\vartheta(X;\ Y;\ 0) = \vartheta_0. \qquad (8)$$

The authors of communications [5–7], having taken the idea from [1], were able to derive approximate formulas to relate the temperatures of bodies with finite dimensions in the case of radiative heat exchange. The particular value of these approximation formulas lies in the fact that they have demonstrated the possi-



Heating of square bar by radiant heat at Ki = 0.5; Θ_0 = 0.15; 1) fin temperature; 2) side temperature; 3) center temperature (curves, computer data; points, calculation according to (10)).

Fo	Ki = 0.3, Θ_0 = 0.15					
	Computer data				Results [Eq.(10)]	% of divergence
	$\Theta(1;Y;Fo)$	$\Theta(1;1;Fo)$	$\Theta(X;1;Fo)$	$\Theta(0;0;Fo)$	$\Theta(0;0;Fo)$	
0,002	0.1586	0,1682	0,1586	0,1500	0,1500	0,00
0,006	0,1714	0,1937	0,1714	0,1500	0,1500	0,00
0,010	0,1800	0,2110	0,1800	0,1500	0,1500	0.00
0,042	0,2153	0,2814	0,2153	0,1500	0,1500	0,00
0,092	0,2510	0,3490	0,2510	0,1521	0,1521	0,00
0,2	0,3171	0,4457	0,3171	0,1853	0,1860	0,38
0,4	0,4324	0,5671	0,4324	0,2891	0,2900	0,31
0,6	0,5417	0,6654	0,5417	0,4032	0,4080	1,19
0,8	0,6389	0,7463	0,6389	0,5114	0,5160	0,90
1,2	0,7907	0,8618	0,7907	0,6957	0,6980	0,33

bility of applying all of the concepts stated in [1] even to the case of radiation heating.

In their writings, the authors of [5–7] proceeded on the basis of the first form of the heat-distribution law for bodies of finite dimensions. For similar purposes it is apparently wiser to rely on expressions corresponding to the second form of the distribution law. Using the substitution.

$$\vartheta(X;\ Y;\ Fo) = \frac{1}{2}[\mathrm{Arth}\,\Theta(X;\ Y;\ Fo) + \mathrm{arctg}\,\Theta(X;\ Y;\ Fo)], \tag{9}$$

it is possible, in a certain approximation [8], to replace Eqs. (5)–(8) by the following system:

$$\frac{\partial\Theta(X;\ Y;\ Fo)}{\partial Fo} = \frac{\partial^2\Theta(X;\ Y;\ Fo)}{\partial X^2} + \frac{R_1^2}{R_2^2}\frac{\partial^2\Theta(X;\ Y;\ Fo)}{\partial Y^2},$$

$$\frac{\partial\Theta(0;\ Y;\ Fo)}{\partial X} = \frac{\partial\Theta(X;\ 0;\ Fo)}{\partial Y} = 0,$$

$$\frac{\partial\Theta(1;\ Y;\ Fo)}{\partial X} = \mathrm{Ki}_1[1-\Theta^4(1;\ Y;\ Fo)],$$

$$\frac{\partial\Theta(X;\ 1;\ Fo)}{\partial Y} = \mathrm{Ki}_2[1-\Theta^4(X;\ 1;\ Fo)],$$

$$\Theta(X;\ Y;\ 0) = \Theta_0$$

This system describes the process of heating a rectangular or square beam by radiated heat. According to law (4) and substitution (9), we will now have

$$[\mathrm{Arth}\,\Theta(X;\ Y;\ Fo) + \mathrm{arctg}\,\Theta(X;\ Y;\ Fo)] = [\mathrm{Arth}\,\Theta(1;\ Y;\ Fo) + \mathrm{arctg}\,\Theta(1;\ Y;\ Fo)] + [\mathrm{Arth}\,\Theta(X;\ 1;\ Fo) + \mathrm{arctg}\,\Theta(X;\ 1;\ Fo)] - [\mathrm{Arth}\,\Theta(1;\ 1;\ Fo) + \mathrm{arctg}\,\Theta(1;\ 1;\ Fo)]. \tag{10}$$

Equation (10) gives the approximate relationship between the temperatures in radiation-heated bodies of finite dimensions. Using this equation, we can determine the temperature of any point within the body from the known temperatures at the surface. Here there is also no need to know the thermal conductivity, the heat capacity, nor the density of the material. Equation (10), just as law (4), is valid for all stages of the heating process (including the initial and ordered periods). And it is only because of the approximate nature of substitution (9) that the use of (10) is limited to the values of the Kirpichev radiation number (less than 0.6) (see the figure and the table). In any event, the calculation error is reduced with a reduction in the Kirpichev number, an increase in the dimensionless initial temperature, and with increasing distance between the point under consideration and the center of the body.

A formula such as (10) can be derived for a parallelepiped and a short cylinder.

Thus on the basis of the laws governing the distribution of heat in bodies of finite dimensions we can derive approximate relationships which are similar to these in terms of their significance and which are applicable to the conditions of radiative heat exchange at the boundaries.

NOTATION

$\Theta = T/T_m$ is the dimensionless temperature; X and Y are dimensionless coordinates; $Fo = a\tau/R^2$ is the Fourier number; $Ki = q_m R/\lambda T_m$ is the Kirpichev number.

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28 December 1967

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